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2007 J. Phys.: Condens. Matter 19 125212

(http://iopscience.iop.org/0953-8984/19/12/125212)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **19** (2007) 125212 (19pp) doi:10.1088/0953-8984/19/12/125212

Spin-charge gauge approach to metal–insulator crossover and transport properties in high-T_c cuprates

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Received 22 December 2006
Published 6 March 2007
Online at stacks.iop.org/JPhysCM/19/125212

Abstract
The spin-charge gauge approach to the metal–insulator crossover (MIC) and other anomalous transport properties in high-T_c cuprates is briefly reviewed. A $U(1)$ field gauging the global charge symmetry and an $SU(2)$ field gauging the global spin–rotational symmetry are introduced to study the two-dimensional t–J model in the limit $t \gg J$. The MIC, as a clue to the understanding of the 'pseudogap' (PG) phase, is attributed to the competition between the short-range antiferromagnetic order and dissipative motion of charge carriers coupled to the slave-particle gauge field. The composite particle formed by binding the charge carrier (holon) and spin excitation (spinon) via the slave-particle gauge field exhibits a number of peculiar properties, and the calculated results are in good agreement with experimental data for both PG and 'strange metal' phases. Connections to other gauge field approaches in studying the strong correlation problem are also briefly outlined.

1. Introduction

The discovery of high-temperature superconductivity 20 years ago (Bednorz and Müller 1986) has posed a great challenge to condensed matter physicists. It is true that there is no consensus yet on theoretical interpretation of the normal state properties and superconducting mechanism for these cuprates. However, enormous progress has been made in material sample preparation, experimental probes of various physical properties using the finest tools and theoretical studies from many different points of view. Although physicists working in this field still have different opinions, most researchers would more or less accept the following scenario:

These cuprates share a layered structure incorporating one or more copper–oxygen planes. The parent compounds (e.g. La_2CuO_4) contain one electron per site which should be metals according to the band theory, but are indeed Mott insulators due to strong Coulomb repulsion. Upon doping beyond a certain concentration (a few per cent) these compounds become superconducting. Meanwhile, in the temperature range above the superconducting transition (usually called the pseudogap (PG) 'phase' on the underdoped side and strange metal (SM) 'phase' on the optimal, or slightly overdoped, side), various physical properties, including optical, magnetic and transport, are anomalous and cannot be described by the standard Landau Fermi liquid (FL) theory. Most people believe that these unusual properties are due to correlation effects in doped Mott insulators. One has to first study these properties using simple models, like the Hubbard model, or the t–J model in the strong coupling limit, to see whether the new physics in the PG and SM phases beyond the FL theory can be essentially understood. Surely there are other effects, like inhomogeneities and electron–phonon interactions, which we have not considered in this paper.

The strong-correlation approach was pioneered by Anderson (1987), reviving his earlier work on frustrated spin-$1/2$ systems at triangular lattices (Anderson 1973, Fazekas and Anderson 1974). Instead of antiferromagnetic (AF) long-range order (LRO), a spin liquid ground state consisting of singlet resonant valence bonds (RVB) was considered. Using the Gutzwiller-projected BCS wavefunction Anderson argued that this RVB state is naturally evolving into a superconducting state upon doping. Later, the spin-charge separation concept (Baskaran *et al* 1987, Kivelson *et al* 1987, Zou and Anderson 1988) was introduced, with a spinon carrying the spin degree of freedom and a holon carrying the charge degree of freedom. Using the slave-boson mean field theory (MFT) (Kotliar and Liu 1988) and Gutzwiller approximation (Zhang *et al* 1988) a d-wave superconductivity and pseudogap state were predicted using the RVB approach. The gauge field fluctuations implementing the single-occupancy constraints beyond the slave-boson MFT were considered by a number of authors (Baskaran and Anderson 1988, Ioffe and Larkin 1989, Nagaosa and Lee 1990). Very recently, a comprehensive review on the strong-correlation approach to high-T_c (HTS) cuprates, focusing on the gauge theory approach, was provided by Lee *et al* (2006).

For the last few years we have been developing a different gauge approach to attack the HTS problem, using a $U(1)$ field to gauge the global charge symmetry and an $SU(2)$ field to gauge the global spin symmetry (Marchetti *et al* 1998, 2000, 2001, 2004a, 2004b, 2005). Unlike the slave-boson approach, the charge degree of freedom in our approach is carried by a fermion, while the spin degree of freedom is represented by a boson. We found this approach to be very efficient in treating the most pronounced phenomenon in the PG phase, namely the metal–insulator crossover (MIC) as well as various transport, optical and magnetic properties. In this short review we will outline the basic ideas of our approach and summarize our main results in comparison with experiments. We will attempt to make the presentation more readable and emphasize the physical interpretation, referring readers to original papers for technical details.

2. Metal–insulator crossover as a clue to the HTS problem

The PG phase was anticipated by the RVB approach and has been studied very thoroughly in an enormous amount of experimental and theoretical work which has been very well summarized in recent reviews (Timusk and Statt 1999, Norman and Pepin 2003, Lee *et al* 2006). In our view the most spectacular phenomenon of the PG phase is the MIC observed in underdoped, non-superconducting cuprates in the absence of magnetic field and a similar phenomenon in superconducting samples when an applied strong magnetic field suppresses the superconductivity.

First of all, this MIC is a rather universal phenomenon. A minimum in resistivity (around 50–100 K) and a crossover from metallic conductivity $\frac{d\rho}{dT} > 0$ at high temperatures to insulating behaviour $\frac{d\rho}{dT} < 0$ at low temperatures has been observed in heavily underdoped LSCO

(Takagi *et al* 1992, Keimer *et al* 1992, Ando *et al* 2001, 2004), non-superconducting $Bi_{2+x}Sr_{2-y}CuO_{6\pm\delta}$ (Fiory *et al*), non-superconducting YBCO (Wuyts *et al* 1996, Trappeniers *et al* 1999, Ando *et al* 1999, 2004) and La-doped Bi-2201 (Ono and Ando 2003). It has also been observed in electron-underdoped NCCO (Onose *et al* 2001) and PCCO (Fournier *et al* 1998).

Moreover, such a MIC has also been observed in a number of superconducting samples when a strong magnetic field suppresses the superconductivity, including LSCO (Ando *et al* 1995, 1996a, Boebinger *et al* 1996), La-doped Bi-2201 (Ando *et al* 1996a, 1996b, Ono *et al* 2000), electron-doped PCCO (Fournier *et al* 1998) as well as in Zn-doped YBCO (Segawa and Ando 1999).

This phenomenon has not had much attention so far, because many people attributed it to localization due to disorder. However, that interpretation is at odds with the following facts: (1) Including higher-doping samples exhibiting MIC in strong magnetic fields, the estimate for $k_F\ell$, where k_F is the Fermi momentum and ℓ the mean free path, ranges from 0.1 to 25 at the MIC, i.e. from far below to far above the Ioffe–Regel limit ($k_F\ell \sim 1$), characterizing the MIC as being due to disorder localization. (2) In LSCO samples with a–b in-plane anisotropy the MIC temperature of ρ_a is different from that of ρ_b (Dumm *et al* 2003), contradicting the 'unique' localization temperature, characteristic of the (at least standard) theory of localization. (3) A universality of suitably normalized resistivity (Wuyts *et al* 1996, Konstantinovic *et al* 2000) has been observed in terms of T/T^*, where T^* is roughly proportional to T at the MIC (T_{MIC}) and can be identified as the PG temperature. All the above features are very difficult, if not impossible, to explain from the localization viewpoint. On the other hand, as discussed below, the spin-charge gauge approach provides a rather natural interpretation for these unusual features (Marchetti *et al* 2004a, 2004b).

Furthermore, once the MIC is recognized as an intrinsic property of cuprates, it is very difficult in conventional theory to reconcile the insulating behaviour at low temperatures with the presence of a finite Fermi surface (FS), as shown by angle-resolved photoemission spectroscopy (ARPES) (see, e.g., Damascelli *et al* 2003). Meanwhile, these two seemingly contradictory phenomena can be easily accommodated in a slave-particle gauge theory, due to the Ioffe and Larkin (1989) rule, stating that the inverse conductivity of the electron is the sum of the inverse conductivity of the spinon and the inverse conductivity of the holon. This non-standard feature can be intuitively understood as a consequence of the gauge string binding spinon to holon: the velocity of the electron is determined by the slowest (not the fastest!) among spinon and holon. Then, if the fermionic excitation, spinon or holon depending on the approach, has a FS and therefore a metallic resistivity vanishing at $T \sim 0$, the electron can still have a metallic/insulating behaviour at low T if the bosonic excitation does. If the leading contribution comes from the boson, without a detailed reference to FS, this can qualitatively explain the universality quoted in point (3) above.

In our view the MIC *is the clue to the understanding of the PG phase*. The MIC in underdoped cuprates in the absence of magnetic field and the MIC in superconducting samples when a strong magnetic field suppresses superconductivity *is the same phenomenon with the same origin*: as an outcome of competition between the AF short-range order (SRO) and the dissipative motion of the charge carriers. We start from the Mott insulating state showing AF LRO. Upon doping beyond a certain threshold the AF LRO is destroyed, being replaced by SRO, characterized by an AF correlation length ξ. Since holes distort the AF background and their average distance $\sim\delta^{-1/2}$, intuitively, $\xi \approx \delta^{-1/2}$, where δ is the doping concentration. This has been confirmed by neutron scattering experiments (Keimer *et al* 1992). Our theoretical treatment proves $\xi \approx (\delta|\ln\delta|)^{-1/2}$, providing the first length scale, while the corresponding energy scale is the spin excitation (spinon) gap $m_s = J(\delta|\ln\delta|)^{1/2}$, where J is the AF exchange interaction. A competing factor is the diffusive motion of the charge carriers with characteristic energy $\sim T m_{\mathrm{h}}$, where T is the temperature, while $m_{\mathrm{h}} \sim \delta/t$ is the effective mass of the charge carrier (holon) in the PG phase and t is the hopping integral. The corresponding length scale is the thermal de Broglie wavelength $\lambda_{\mathrm{T}} \sim (T\delta/t)^{-1/2}$. At low temperatures, $\xi \lesssim \lambda_{\mathrm{T}}$, the AF SRO dominates and the charge carriers become weakly localized (not exponentially) showing insulating behaviour. We would like to emphasize that this 'peculiar localization' is mainly due to interaction rather than disorder, and it is also different from the standard Mott insulator which comes entirely from the Coulomb repulsion. On the contrary, at high temperatures, $\xi \gtrsim \lambda_{\mathrm{T}}$, the diffusive motion of charge carriers prevails, exhibiting metallic conductivity. Therefore the competition between the real part of the spinon 'self-energy', the mass gap, and the imaginary part, the dissipation, gives rise to this spectacular phenomenon: MIC.

It turns out that a number of experimental observations in the PG phase can also be explained by the competition of these two factors and the 'composite' nature of low-energy excitations. Roughly speaking, the spin (spinons) and charge (holons) excitations in the PG phase behave like 'separate particles' in their scattering against gauge fluctuations, which renormalizes their properties and dominates the in-plane transport phenomena. However, at small energy–momentum scale the gauge field binds spinon and anti-spinon into magnon 'resonance'. Similarly, spinon and holon are bound into electron 'resonance' with non-Fermi-liquid properties, showing up in ARPES experiments as a 'quasiparticle peak' and responsible for interlayer transport. In particular, the 'composite' nature of particles is exhibited in the anomalously large spectral weight away from the quasi-particle peak seen in ARPES (see, e.g., Nagaosa and Lee 1990, Lee and Nagaosa 1992, Laughlin 1997, Orgad *et al* 2001). In a sense, the low-energy excitations in these strongly correlated materials are no longer well-defined quasi-particles of FL theory, but rather these loosely bound 'composite particles', taken as a first step going beyond the well-established condensed matter physics paradigm.

3. Spin-charge decomposition: statistics and symmetries

Our basic assumption is that the low-energy physics of the Cu–O layers in HTS cuprates can be described qualitatively by a two-dimensional t–J model:

$$H = P_{\mathrm{G}} \sum_{\langle ij\rangle} \left(-t \sum_{\alpha} c^*_{\alpha i} c_{\alpha j} + J(\vec{S}_i \cdot \vec{S}_j - \tfrac{1}{4} n_i n_j) \right) P_{\mathrm{G}}, \tag{3.1}$$

where i correspond to Cu sites, $\alpha = 1, 2$ is the spin index and P_{G} denotes the Gutzwiller projection, eliminating the double occupancy. Numerical simulations yield $t \simeq 0.4$ eV, $J \simeq 0.13$ eV which we use in calculations of physical quantities. The qualitative behaviour of the low-energy physics in the c-direction is obtained by adding an interlayer hopping term.

Once the t–J model is accepted, a key question is how to handle the Gutzwiller projection. A straightforward way is either to use numerical techniques (Gros 1988, 1989, Paramekanti *et al* 2001, 2003) or MFT (Zhang *et al* 1988, Anderson *et al* 2004). Another way is to decompose the electron into holon and spinon:

$$c_\alpha = h s_\alpha. \tag{3.2}$$

A simple counting of degrees of freedom (dof) shows that the decomposition (3.2) has an unphysical redundancy, i.e. a local gauge symmetry. In fact, the electron has four dof (c_α, c^*_α), while the spinon s_α being still of two components, also has four dof, and the holon h, being spinless, has two dof. One has to impose the constraint of no double occupancy, which in slave-particle formalism becomes holonomic. Then the redundant dof ($4 + 2 - 1 = 5$) corresponds to a local phase factor by which the holon can be multiplied, while the spinon is multiplied by its inverse: a $U(1)$ gauge symmetry. It is very useful to make manifest the 'hidden' gauge symmetry by introducing an emergent $U(1)$ gauge field, coupling spinons to holons.

3.1. Different choices of spin-charge decomposition

Accepting the spin-charge decomposition as a good technique to explore the low-energy physics of cuprates, one still has several options for its implementation. The first choice concerns the statistics of spinon and holon. If no approximations are made all these choices are equivalent and a proof of this equivalence for some of them is given in Fröhlich and Marchetti (1992). However, as soon as approximations are made, like mean field (MF), they become intrinsically different. Standard choices are: s_α fermion and h hard-core boson or s_α hard-core boson and h fermion. These choices are at the basis of the traditional slave-boson (Zou and Anderson 1988, Nagaosa and Lee 1990, Lee and Nagaosa 1992) and slave-fermion (Arovas and Auerbach 1988, Yoshioka 1989, Dorey and Mavromatos 1991) approaches. To describe PG the more sophisticated approach to the slave-boson pursued in Wen and Lee (1996), Lee *et al* (1998) is obtained by replacing a single boson h with a pair of bosons h_α, enhancing the slave-particle symmetry from $U(1)$ to $SU(2)$. A slave-fermion version of this is discussed in Farakos and Mavromatos (1998).

Moreover, in 2D there are also other 'anyon' statistics available, in particular the 'semion statistics' advocated originally by Laughlin (1988), in which an interchange of the fields produces a factor $e^{\pm i\frac{\pi}{2}}$. The product of two semions is still a fermion: $(e^{\pm i\frac{\pi}{2}})^2 = -1$. Notice also that for semions a kind of 'hard-core exclusion' holds (Fröhlich and Marchetti 1992), here ensured by P_G.

To adequately describe the FS of cuprates determined by ARPES (Damascelli *et al* 2003) is an important guideline in making choices: for optimally doped materials one finds a FS agreeing with band calculations. This is clearly compatible with s_α being a fermion, as in the slave-boson approach, but it is troublesome for the slave-fermion approach because h being spinless the MF Fermi momentum is expected to be doubled w.r.t. to the electron case (Lee *et al* 2006). For a semion, although the situation is not completely elucidated (Wu 1994, Haldane 1994, Chen and Ng 1995), it appears that a kind of generalized Pauli principle holds, related to Haldane statistics (Haldane 1991), stating that for low T one can accommodate at most two (spinless) semions in the same momentum state and the semion distribution function is approximately twice the fermion distribution function. Hence a gas of spinless semions of finite density should give a 'FS' coinciding with that of spin $\frac{1}{2}$ fermions of the same density. We take here this generalized Pauli principle for granted.

3.2. Hint from the 1D t–J model

The 1D t–J model in the limit $J/t \searrow 0$ has been solved exactly by the Bethe ansatz method (Ogata and Shiba 1990), while the correlation functions have been calculated using conformal field theory (CFT). Applying the spin-charge decomposition approach we have reproduced the main results of these techniques within a kind of MFT (Marchetti *et al* 1996). A coordinate representation of the solution exhibits the following features:

(1) Spin-charge separation: the charge and the spin dof are characterized by different physical behaviours, in particular their velocities are different $v_s \neq v_c$ (Luttinger liquid).
(2) The low-energy physics of the charge dof is described by a free spinless fermion (holon), while the low-energy physics of the spin dof is described by an spin-1/2 AF Heisenberg model in a squeezed spin chain, obtained by omitting the unoccupied sites of the original 1D lattice, where the spins are given by $\vec{S} = S^*_\alpha \frac{\vec{\sigma}_{\alpha\beta}}{2} S_\beta$ with S_α a 'renormalized' gapless

spinon. S_α can be identified as a Gutzwiller projected fermion in the squeezed chain (Marchetti *et al* 1996). The scaling limit of such a model is the $O(3)$ nonlinear σ-model with Θ vacua at $\Theta = \pi$, the value when gapless excitations appear in 1D.

(3) The electron field is a product of a holon and a spinon field together with a non-local 'string phase': $c_{\alpha x} = h_x \exp[\frac{\mathrm{i}}{2}\Sigma_{j<x} h_j^* h_j] S_{\alpha x}$. It can be shown (Marchetti *et al* 1996) that both $S_{\alpha x}$ and

$$h_x \mathrm{e}^{\mathrm{i}\frac{\pi}{2}\Sigma_{j<x} h_j^* h_j} \tag{3.3}$$

describe semions. One can prove that S_α is just a MF approximation of a non-local spinon field with a 'string phase' analogous to that in (3.3), constructed out of bosonic spinons s_α, and its explicit expression is given in Marchetti *et al* (1996). The semion statistics is crucial for obtaining the correct correlation functions in the scaling limit (Marchetti *et al* 1996) (see also Ha and Haldane 1994).

One should note that this statistics is compatible with an optimization of energy: in the semion approach the spinon s_α appears in the hopping term in the Affleck–Marston (AM; Affleck and Marston 1988) form AM $= \sum_\alpha s_{\alpha i}^* s_{\alpha j}$, while in the AF-Heisenberg term in the RVB form, RVB $= \sum_{\alpha\beta} \epsilon_{\alpha\beta} s_{\alpha i} s_{\beta j}$, as $|\mathrm{RVB}|^2$. This occurs because for a holon-empty site j the spinon contribution turns out to be given by

$$\begin{pmatrix} \tilde{s}_{1j} \\ \tilde{s}_{2j} \end{pmatrix} \equiv \begin{pmatrix} s_{1j} & -s_2^* \\ s_{2j} & s_{1j}^* \end{pmatrix} \sigma_x^j \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \tag{3.4}$$

where the spin flip induced by σ_x is due to an emergent AF structure. However, for a holon-occupied site there is an additional spin flip and this is the case for the final site of a hopping link. The peculiar feature of this spin flip occurs only in a $SU(2)$ formalism. The interesting point is that the following identity holds for s_α hard-core boson: $|\mathrm{AM}|^2 + |\mathrm{RVB}|^2 = 1$. Optimization of the t term suggests $|\langle \mathrm{AM} \rangle| = 1$, but due to the previous identity this choice also optimizes the positive J term, since it corresponds to $|\langle \mathrm{RVB} \rangle| = 0$.

From (3.3) one can see that a semion can be constructed out of fermions (and in similar way also from hard-core bosons) through a generalization of the Jordan–Wigner transformation (Fradkin 1991) with an exponent which is half of the standard one. The continuum analogue of (3.3) is

$$h(x) \mathrm{e}^{\mathrm{i}\frac{\pi}{2}\int H(x-y) h^*(y) h(y)\,\mathrm{d}y} \tag{3.5}$$

where $H(x)$ is the Heaviside step function. The representation (3.5) makes it clear that the 'statistics transmutation' (from fermion to semion) in 1D is obtained by adding an exponential of a 'step function' or 'kink' average of the density.

3.3. Generalization to two dimensions

If we accept the suggestion coming from the 1D model for the statistics of holon and spinon, one should search for a semionic representation of the electron field in 2D. A semion field in 2D is constructed out of a fermion as in (3.5) by replacing the 1D 'kink' $H(x-y)$ with a 2D 'vortex' $\arg(\vec{x}) \equiv \arctan(\frac{x_2}{x_1})$ in the average of the density.

It is convenient to rewrite the resulting exponential in terms of a gauge field

$$\vec{B}(y) = \frac{1}{2}\int \vec{\nabla} \arg(\vec{y} - \vec{z}) h^* h(z, y^0)\,\mathrm{d}^2 z \tag{3.6}$$

as $\exp[\mathrm{i}\int_x^\infty \vec{B}(y) \cdot \mathrm{d}\vec{y}]$.

To show that indeed $\vec{B} \equiv \{B_\mu\ \mu = 1, 2\}$ is the gauge field associated with a vortex let us compute its field strength, using $\vec{\nabla} \times \vec{\nabla} \arg(x - y) = \delta(x - y)$ we get

$$\vec{\nabla} \times \vec{B}(y) = \tfrac{1}{2}\rho(y), \tag{3.7}$$

where ρ is the density of h. (Notice that from $\vec{\nabla}\cdot\vec{\nabla} \arg(\vec{x}-\vec{y}) = 0$ it follows $\vec{\nabla}\cdot\vec{B} = 0$, i.e. $\vec{B}$ is in the Coulomb gauge.) Equation (3.7) is the same as that appearing in the fractional quantum Hall effect (FQHE) (see, e.g. Fradkin 1991) replacing $\frac{1}{2}$ with $\frac{1}{\nu}$ where ν is the filling at the plateau. Hence the vortices appearing in (3.6) are analogous to those introduced by Laughlin in the FQHE and in fact a semionic representation of the electron was advocated by him (Laughlin 1988) in the early days of HTS.

Actually the physics here is dual to that of FQHE: there the flux was integer and the charge fractional, while here it is the opposite. The above ideas have been made precise, within the spin-charge gauge approach, in Marchetti *et al* (1998), where the following decomposition of the electron has been proved, in terms of a fermionic holon and a bosonic spinon s_α satisfying the Gutzwiller constraint $\sum_\alpha s^*_\alpha s_\alpha = 1$:

$$c_x \sim h_x \mathrm{e}^{\mathrm{i}\int_x^\infty \vec{B}(y)\cdot \mathrm{d}\vec{y}} P(\mathrm{e}^{\mathrm{i}\int_x^\infty \vec{V}(y)\,\mathrm{d}\vec{y}})_{\alpha\beta} s_{\beta x}, \tag{3.8}$$

where $\vec{B}$ and $\vec{V}$ are $U(1)$ and $SU(2)$ gauge fields, respectively, and P is a path-ordering needed for the non-Abelian nature of V_μ. In (3.8) we have

$$\vec{B}(y) = \tfrac{1}{2}\sum_j \vec{\nabla} \arg(\vec{y} - j)(1 - h^*_j h_j(y^0)), \tag{3.9}$$

$$\vec{V}(y) = \sum_{a=1}^{3} \sigma_a \vec{\nabla} \arg(\vec{y} - j)(1 - h^*_j h_j(y^0))(\tilde{s}_{\alpha j}\sigma^a_{\alpha\beta}\tilde{s}_{\alpha j}(y^0)) \tag{3.10}$$

with $\tilde{s}_{\alpha j} = s_{\alpha j}$ if j is on the even Néel sublattice and $\tilde{s}_{\alpha j} = \epsilon_{\alpha\beta} s^*_{\beta j}$ if j is on the odd Néel sublattice (compare with (3.4)). Here an AF structure emerges formally and the basic related assumption is that fields s_α have a good continuum limit, but not $\tilde{s}_\alpha$. The physical spin turns out to be

$$\vec{S}_j \simeq \tilde{s}^*_{\alpha j}\vec{\sigma}_{\alpha\beta}\tilde{s}_{\beta j} = (-1)^{|j|} s^*_{\alpha j}\vec{\sigma}_{\alpha\beta} s_{\beta j}, \tag{3.11}$$

where $|j| = 0(1)$ for a site in the even (odd) Néel sublattice, so that the AF ordering naturally evolves as $\delta \searrow 0$ to the LRO of the 2D AF Heisenberg model, in agreement with experiments in undoped cuprates. In (3.8) both $h_x \mathrm{e}^{\mathrm{i}\int_x^\infty \vec{B},\mathrm{d}\vec{y}}$ and $(P\mathrm{e}^{\mathrm{i}\int_x^\infty V\,\mathrm{d}\vec{y}})_{\alpha\beta} s_{\beta x}$ are semion fields.

3.4. Our choice of mean field approximation

Where do the $U(1)$ and the $SU(2)$ groups of (3.9), (3.10) come from? They are just the global charge-$U(1)$ and spin-$SU(2)$ symmetry (not to be confused with the $SU(2)$ slave-particle symmetry of Wen and Lee 1996, Lee *et al* 1998) of the t–J model. In Marchetti *et al* (1998) it is shown that if these spin-charge symmetries are made local by introducing a coupling with a $U(1)$ field $\vec{B}$ and an $SU(2)$ field $\vec{V}$ given by (3.9), then the obtained 'spin-charge gauged t–J model' is strictly equivalent to the original t–J model[4]. Of course, as soon as MF approximations are made, this approach becomes distinct from the slave boson or slave fermion, for example, becoming a new 'slave-semion' approach. A variant of this is obtained by replacing the $SU(2)$ of spin with the $U(1)$ subgroup unbroken in the AF phase. This is the choice made by Weng and collaborators (Weng *et al* 2000, Weng 2003).

However, in the 'semionic' approach it is too hard to keep charge, spin and slave-particle symmetries all exact and an approximation has been made to perform explicit computations.

[4] In path integral formalism this is obtained within a Chern–Simons theory.

In the spin-charge gauge approach pursued by us, the spin-charge gauge symmetry is treated in a sort of MFT while the slave-particle symmetry is kept exact. An opposite choice has been adopted by Weng (Weng *et al* 2000, Weng 2003), i.e. the charge and the (reduced) spin symmetries are kept exact as much as possible, but the slave-particle symmetry is treated in MFT.

The second major choice in the MF approaches is the spinon order parameter: if AM-like, i.e. $\langle \sum_\alpha s^*_{\alpha i} s_{\beta j} \rangle$ (Affleck and Marston 1988) and nonvanishing, it leaves unbroken the global slave-particle symmetry but breaks the global spin $SU(2)$; if RVB-like, i.e. $\langle \sum_{\alpha\beta} \epsilon_{\alpha\beta} s_{\alpha i} s_{\beta j} \rangle$ (Wen and Lee 1996, Lee *et al* 1998) and non-vanishing, it breaks the global slave-particle symmetry but preserves the global spin $SU(2)$.

In the standard slave-boson approach to PG one has $\langle \mathrm{RVB} \rangle \neq 0$, though in the $SU(2)$–slave-boson approach a $U(1)$ subgroup is unbroken. A kind of slave-boson approach where in the PG one has $\langle \mathrm{AM} \rangle \neq 0$ is pursued in Feng *et al* (2004).

The choice of the MF order parameter has relevant physical consequences, because the Ioffe and Larkin (1989) rule does not appear naturally compatible with a MF in which the slave-particle symmetry is broken.

Since in the spin-charge gauge approach we have $\langle \mathrm{AM} \rangle \neq 0$, but $\langle \mathrm{RVB} \rangle = 0$ in MF, in both PG and SM 'phases' the Ioffe–Larkin rule holds. Furthermore since in MF $|\langle \mathrm{AM} \rangle| = 1$, the semion approach seems to optimize both the t and the J terms, as discussed in 1D (but here a rigorous proof is lacking) in compatibility with an AF structure.

4. Spin-charge gauge approach: the key ingredients

To summarize, the key physical ingredients of the spin-charge gauge approach of MFT to characterize the 'normal state' within the t–J model are: the Gutzwiller projection, tackled with spin-charge decomposition, the two-dimensionality needed for semion statistics, finally leading to a spinon gap, and the bipartite lattice structure needed for both AF and the π–0 flux crossover, which is also a peculiar 2D phenomenon.

4.1. Energy optimization and the π–0 flux crossover

Here we focus on the π–0 flux crossover. A key step is a 'semiclassical' treatment of the interaction of $\vec{B}$ in the holon hopping term $t h^*_i h_j U_{\langle ij \rangle}$, where $U_{\langle ij \rangle}$ is a complex Abelian gauge field given by

$$U_{\langle ij \rangle} = \mathrm{e}^{\mathrm{i} \int_i^j \vec{B}(y) \cdot \mathrm{d}\vec{y}} s^*_{\alpha i} (P \mathrm{e}^{\mathrm{i} \int_i^j \vec{V}(y) \cdot \mathrm{d}\vec{y}})_{\alpha\beta} s_{\beta j}. \tag{4.1}$$

If p denotes a plaquette and $\langle ij \rangle \in \partial p$ the links in the boundary of p, the argument of $\prod_{\langle ij \rangle \in \partial p} U_{\langle ij \rangle}$ plays the role of a magnetic flux.

It has been rigorously proved by Lieb (1994) that at half-filling ($\delta = 0$) the optimal configuration for a magnetic field on a square lattice in 2D has a flux π per plaquette at arbitrary temperature. On the other hand, it is well known that at low densities and high temperatures the optimal configuration has 0 flux per plaquette. At $T = 0$ it has been proven that the ground state energy has a minimum corresponding to one flux quantum per spinless fermion (Bellisard and Rammal 1990). Numerical simulations (Qin 2000) suggest that increasing T gives rise to a competition between these minima and at sufficiently high T only 0 and π flux survive. We expect that the perturbation introduced by the J term in the t–J model changes the boundary, but not the essence of the phenomenon of the $\pi \longrightarrow 0$ crossover itself, while further enhancing the 'disorder' lowers the temperature where only π and 0 flux states survive.

In the spin-charge gauge MFT it is assumed that the region of π flux of U corresponds to PG and that of 0 flux to SM; thus the 'melting of the π-flux lattice' underlines the physics of crossover between the two 'phases', which in particular produces a modification in the structure of the FS.[5]

The optimization considerations discussed above suggest a MF approximation for $\vec{B}$ given by

$$\vec{B}(y) \longrightarrow \vec{B}_{\rm MF}(y) = \tfrac{1}{2} \sum_j \vec{\nabla} \arg(y-j), \tag{4.2}$$

which gives flux π per plaquette. Furthermore, to obtain the $\pi \longrightarrow 0$ crossover in the MF treatment of the holon hopping one imposes, by a suitable spinon redefinition, that

$$\prod_{\langle ij\rangle \in \partial p} (s^*_{\alpha i}(P e^{i\int_i^j \vec{V}(y)\cdot d\vec{y}})_{\alpha\beta} s_{\beta j}) \sim \begin{cases} 1 & \text{PG} \\ e^{i\pi} & \text{SM} \end{cases}$$

so that the effect of $\vec{B}_{\rm MF}$ on holons in SM is cancelled by the spinon contribution. The unbalanced presence of $\vec{B}_{\rm MF}$ in PG produces the Hofstadter phenomenon: it converts the spinless holon h of SM with dispersion

$$\omega \sim 2t[(\cos k_x + \cos k_y) - \delta] \tag{4.3}$$

and Fermi momenta $k_{\rm F} \sim 1-\delta$, i.e. a 'large FS' roughly consistent with band calculation, into a pair of 'Dirac fields', with pseudospin index corresponding to the two Néel sublattices and dispersion relation:

$$\omega \sim 2t(\sqrt{\cos^2 k_x + \cos^2 k_y} - \delta) \tag{4.4}$$

restricted to the magnetic Brillouin zone, thus yielding a 'small FS' centred at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ with Fermi momenta $k_{\rm F} \sim \delta$. As we will see, many crossover phenomena from PG to SM in transport physics are due to this change of holon Fermi momenta.

4.2. Generation of the spinon mass

Comparing (4.2) and (3.9) one can see that the MF approximation for $\vec{B}$ consists in neglecting fluctuations of h; it is therefore reasonable in a MF treatment of $\vec{V}$ to neglect the spinon fluctuations, i.e.

$$\vec{V}(y) \longrightarrow \vec{V}^a_{\rm MF}(y) = \delta^{a3} \sum_j (-1)^{|j|} \vec{\nabla} \arg(y-j)(1-h^*_j h_j). \tag{4.5}$$

The first term of $\vec{V}^a_{\rm MF}$ can be gauged away, while the second describes spin vortices, centred at the holon positions, of opposite vorticity (or chirality) for holons on two Néel sublattices. These chiral spin vortices are reminiscent of those introduced by Shraiman and Siggia (1988) in analysing the single hole problem, but here their density is finite $\delta > 0$.

Due to their AF nature the spinons in this approach have a low-energy effective action described by a O(3) nonlinear σ-model plus an interaction with spin vortices appearing in $\vec{V}_{\rm MF}$. Due to vortex chirality, the spatial average of the interaction term linear in $\vec{V}_{\rm MF}$ vanishes, hence in MF approximation the leading spinon–vortices interaction term is given at large scales by $\langle \vec{V}^3_{\rm MF} \cdot \vec{V}^3_{\rm MF}\rangle s^*_\alpha s_\alpha$ where $\langle\ \rangle$ denotes spatial average[6]. Naively one expects this average to be proportional to δ, as this is the density of holons, hence of vortices. However, due to the long-range tail of vortices one actually gets (Marchetti *et al* 1998) a logarithmic correction, which,

[5] Let us remark that even in principle not all phenomena usually referred to as 'pseudogap' should fit in this interpretation, but only those referring to the T^* temperature discussed below.

[6] Self-consistently the AF LRO is assumed absent.

we will see later on, is physically relevant. Therefore, as a result of the interaction with vortices, in MF the spinons acquire a mass gap m_s given by

$$m_s^2 \sim \langle \vec{V}^3_{\rm MF} \cdot \vec{V}^3_{\rm MF} \rangle \sim \delta |\ln \delta|. \tag{4.6}$$

The gap generation here is a kind of wave localization phenomenon: the gapless waves (here spinons) interacting with a finite (supercritical) density of impurities (here the vortices), are getting localized, acquiring a gap. The AF massive structure of the spinon in our approach prevents boson condensation at low T, in contrast to the slave-boson approach.

Since the spinon gap has an obvious relation with the AF correlation length, $\xi_{\rm AF}$, its appearance implies that the finite (supercritical) density of vortices converts the long-range AF of the undoped ($\delta = 0$) material into a short-range AF and the derived doping dependence (roughly $\xi_{\rm MF} \sim \delta^{-1/2}$) is consistent with the experimental data in neutron experiments in LSCO (Keimer *et al* 1992). Actually, by including the spinon thermal mass $m_{\rm T}$ of the renormalized classical treatment of the $O(3)$-model, both the order of magnitude and the qualitative δ–T dependence of the neutron data (Keimer *et al* 1992) are reproduced. The T independent term found in experiments which is hard to explain in terms of a standard $O(3)$ model, as in the spin-Fermion approach (Abanov *et al* 2003), is here exactly due to the introduction of the spin vortices, a unique feature of this approach.

4.3. Reizer singularity of the slave-particle gauge field

As already remarked, in slave-particle approaches a slave-particle $U(1)$ gauge field couples spinon and holon. In the spin-charge gauge approach this can be identified with the gauge field, A_μ, of the $O(3)$-model rewritten in CP^1, i.e. the Lagrangian for spinon is

$$L(A, s) \sim \frac{1}{J} \left[|(\partial_\mu - A_\mu) s_\alpha|^2 + m_s^2 s^*_\alpha s_\alpha \right] \tag{4.7}$$

$\mu = 0, 1, 2$. It turns out that $A_\mu \sim \tilde{s}^*_\alpha \partial_\mu \tilde{s}_\alpha$, therefore it is staggered.

The same gauge field appears in the covariant derivative, obtained via Peierls substitution, acting on holons. Integrating out spinons and holons one obtains the low-energy effective action for A_μ. Since s_α is AF and gapful, the spinon contribution is Maxwellian with a thermal mass m_0 for A_0. On the contrary, holons have a finite FS (in both PG and SM) and their contribution to the transverse component exhibits the so-called Reizer singularity (Reizer 1989a, 1989b).

As a result the leading behaviour of the A propagator is: with $i, j = 1, 2$

$$\langle A^{\rm T}_i A^{\rm T}_j \rangle (\omega, q) \sim \left(\delta_{ij} - \frac{q_i q_j}{\vec{q}^2} \right) \left({\rm i}\kappa \frac{\omega}{|\vec{q}|} - \chi \vec{q}^2 \right)^{-1} \tag{4.8}$$

where $\kappa \sim k_{\rm F}$ is the Landau damping and $\chi = \chi_{\rm s} + \chi_{\rm h}$ with $\chi_{\rm s} \sim \frac{J}{6\pi m_s}$, $\chi_{\rm h} = \frac{t}{6\pi k_{\rm F}}$, is the total diamagnetic susceptibility, dominated, at least for small δ, by the holon contribution.

There is a characteristic scale of gauge fluctuations emerging from the Reizer singularity at finite T. It is a kind of anomalous skin depth, derived by assuming as typical energy $\omega \sim T$, with the consequence that the transverse gauge interaction is peaked at $q = Q_{\rm T} = (\frac{\kappa T}{\chi})^{1/3}$.

The Reizer singularity in this approach plays a crucial role in the interpretation of transport properties, and we remark that it is due to the simultaneous appearance of a finite FS and a gap for the bosonic excitations s_α; if these bosons condense it disappears. This is what happens at low temperatures in the slave-boson approach, which thus has the difficulty that the Ioffe–Larkin rule cannot be extended below the holon condensation temperature.

The scalar component A_0 has a low-energy propagator given by

$$\langle A_0 A_0 \rangle (\omega, q) \sim \left(\kappa \left(1 + {\rm i} \frac{\omega}{|\vec{q}|} \right) H(|\vec{q}| - |\omega|) + m_0^2 \right)^{-1}. \tag{4.9}$$

In view of the constant term in (4.9) the interaction mediated by A_0 is short ranged, hence subleading at large distance w.r.t. the interaction mediated by A^{T}, triggered by the Reizer (1989a, 1989b) singularity.

5. The effect of gauge fluctuations on correlation functions

A difficulty in considering the gauge fluctuations is the lack of a 'small parameter' for expansion. Also, a perturbative treatment would be insufficient to get a bound state or a resonance with the electron quantum numbers out of a spinon and a holon, whereas some kind of binding is expected on the basis of the considerations leading to the Ioffe–Larkin rule. A possible way out is to implement the idea of binding using an eikonal approach in studying the hydrogen atom as a bound state of a proton and electron. To analyse the behaviour of correlation functions of physical, hence gauge invariant, fields we apply a kind of eikonal resummation of (transverse) gauge fluctuations (Marchetti *et al* 2004a). This resummation is obtained by treating first A_μ as an external field, expanding the correlation function in terms of first-quantization Feynman paths, then integrating out A_μ to obtain an interaction between paths which is then treated in the eikonal approximation. Finally a Fourier transform is performed to get the retarded correlation function. Further approximations are needed however, especially in the treatment of short-scales, to get the final result; we refer the reader to (Marchetti *et al* 2004a, 2005) for details, but we briefly comment on some interesting features encountered in the calculation of, for example, the magnon correlation function.

5.1. Magnon correlation function

It turns out numerically that for δ, T small (identified with PG) the spatial Fourier transform in the eikonal approximation is dominated by a nontrivial complex $|\vec{x}|$-saddle point, $|x|_{\mathrm{s.p.}} \sim Q_{\mathrm{T}}^{-1}\mathrm{e}^{\mathrm{i}\pi/4}$ due to the effect of gauge fluctuations. The self-consistency requirement for this eikonal approximation yields a region of validity given approximately by $m_{\mathrm{s}}Q_{\mathrm{T}} \lesssim T/\chi \lesssim m_{\mathrm{s}}^2$ (inserting numbers as order of magnitude from a few tens to a few hundred K). The upper bound temperature, T^*, roughly coincides both as an order of magnitude and as δ–T dependence (for low doping) with the pseudogap temperature, slowly decreasing with δ due to a delicate cancellation, where the relevance of the logarithmic correction in (4.6) shows up: $T^* \sim \chi m_{\mathrm{s}}^2 \sim \frac{t}{6\pi\delta}|\delta\ln\delta| \sim \frac{t}{6\pi}|\ln\delta|$. The lower bound temperature T_{sg}, with a maximum at $\delta \sim 0.02$ and then slowly decreasing, is reminiscent of the spin-glass crossover in the strongly underdoped region.

The main effect of the complex saddle point within the above range is to induce a shift in the mass of spinons, $m_{\mathrm{s}} \to M = (m_{\mathrm{s}}^2 - \mathrm{i}cT/\chi)^{1/2}$, where $c \sim 3$ is a constant, thus introducing a dissipation proportional to T. Physically, due to the Ioffe–Larkin rule, electron conductivity is dominated by spinons, and the competition between the mass gap and the dissipation appearing in M is responsible for the MIC of in-plane resistivity and of many other crossovers in PG, as discussed already in section 2.

For sufficiently high δ or T (region identified with SM) one can verify numerically that the saddle point contribution is negligible w.r.t. the contribution of fluctuations around 0 in the range $|\vec{x}| \lesssim Q_{\mathrm{T}}^{-1}$. The result of gauge fluctuations can also be summarized in terms of the appearance of a dissipative term: $m_{\mathrm{s}} \to m_{\mathrm{s}} - \mathrm{i}c'TQ_{\mathrm{T}}/\chi m_{\mathrm{s}}^2$, with $c' \sim 0.1$. In SM, however, the change of k_{F} from δ to $1-\delta$ yields a decrease of diamagnetic susceptibility χ, implying that the thermal de Broglie wavelength is shorter than its PG counterpart. Therefore the spin-gap effects ($\xi_{\mathrm{AF}} < \lambda_{\mathrm{T}}$) are less effective, being confined to very low temperatures. At a pictorial level the existence of a characteristic scale $|\vec{x}| \sim Q_{\mathrm{T}}^{-1}$ in PG seems to indicate that spinon and antispinon in the magnon have this typical size, while in SM Q_T^{-1} is just the typical range of their spatial fluctuations. Our approximate time-Fourier transform involves a UV cutoff λQ_T^{-1} in terms of a parameter λ fixed phenomenologically by comparison with experimental data. It turns out that $\lambda \ll 1$ in PG and in SM the result is self-consistent only if $m_s Q_T^{-1}, \frac{t}{\chi m_s} \lesssim 1$ with $\lambda \simeq 1$ (numerically yielding a range from few tens to few hundred K). The retarded magnon propagator derived in this manner is given by

$$\langle \vec{\Omega} \cdot \vec{\Omega} \rangle(\omega, \vec{q}) \sim \begin{cases} Z_\Omega^{\mathrm{PG}} \dfrac{1}{\omega - 2M} J_0(|\vec{q}|(2Q_T)^{-1} e^{i\pi/4}) & \mathrm{PG} \\ Z_\Omega^{\mathrm{SM}} \dfrac{1}{\omega - 2m_s + i\frac{T}{\chi m_s^2} c' Q_T \lambda} e^{-\frac{|\vec{q}|^2}{a}} \dfrac{1}{a} & \mathrm{SM} \end{cases} \tag{5.1}$$

where J_0 is the Bessel function, $a \sim \frac{T}{\chi m_s} Q_T$ and $\vec{q}$ is measured from the AF wavevector.

For the wavefunction renormalization constant we have $Z_\Omega^{\mathrm{PG}} \sim Q_T^{-1}(k_F M)^{1/2}$ with $k_F \sim \delta$ and $Z_\Omega^{\mathrm{SM}} \sim Q_T^{-2} k_F m_s$ with $k_F \sim 1 - \delta$. Equation (5.1) implies that, within the approximation scheme adopted, the transverse gauge fluctuations couple the spinon–antispinon pair in $\vec{\Omega}$ into a magnon resonance with mass gap

$$m_\Omega \sim 2m_s \tag{5.2}$$

and inverse lifetime

$$\Gamma_\Omega = \begin{cases} \mathrm{Im}\, M \sim T \text{ if } m_s^2 \gg \dfrac{cT}{\chi}, \sim T^{1/2} \text{ if } m_s^2 \sim \dfrac{cT}{\chi} & \mathrm{PG} \\ \dfrac{T}{\chi m_s^2} c' Q_T \lambda \sim T^{4/3} & \mathrm{SM}. \end{cases} \tag{5.3}$$

As briefly discussed below, Γ_Ω^{-1} is proportional to the electron lifetime, which thus always increases as T decreases. How then this is compatible with MIC? The point is that above MIC the increase of the electron lifetime yields an increase of conductivity, but below MIC mobility decreases as T lowers because the spinon can move only through thermal diffusion, induced by gauge fluctuations due to gapless holons, and, because of binding, the electron can move only if the spinon does. Finally we expect that the effect of the neglected residual short-range attraction mediated by A_0 gives a renormalization of the mass gap, without introducing significant qualitative changes.

5.2. Electron propagator

Let us now briefly comment on the electron correlation function. Previously we discussed the effect of gauge fluctuations on the Ω correlation function at large scales, using the first quantization path-integral representation for the massive spinon propagator. An analogous representation is hard to use for the holon correlation function because of the finite density of holons. This representation would in fact contain a series of alternating sign contributions, corresponding to an arbitrary number of closed fermion wordlines, describing the contributions of h-particles in the finite-density ground state (see, e.g., Fröhlich and Marchetti 1992). To overcome this difficulty, we apply a dimensional reduction by means of the tomographic decomposition introduced by Luther and Haldane (Luther 1979, Haldane 1994). To treat the low-energy degrees of freedom we choose a slice of thickness k_F/Λ, with $\Lambda \gg 1$, in momentum space around the FS of the holon. We decompose the slice in approximately square sectors; each sector corresponds to a quasi-particle field in the sense of Gallavotti–Shankar renormalization (Benfatto and Gallavotti 1990, Shankar 1994) (see also Fröhlich *et al* 1994). Each sector is characterized by a unit vector $\vec{n}(\theta)$, pointing from the centre of the FS to the centre of the box, labelled by the angle θ between this direction and the k_x axis. The contribution of each sector can be viewed approximately as arising from a quasi 1D chiral fermion; this avoids the finite FS problem for the path-integral representation discussed above. The paths appearing for a sector are straight lines directed along the Fermi momenta of the sector, with small Gaussian transverse fluctuations. We now multiply this path representation for the holon by that for the spinon, following the decomposition formula (3.8) for the electron treated in MF. Finally we integrate out in the eikonal approximation the transverse gauge fluctuations, proceeding as for the magnon correlation function. The output for the retarded correlation function becomes simple only for momenta on the FS; however, the main features presented here hold in a slab of scale Q_{T} around the FS. On the FS the result can be summarized as follows:

$$G^R(\omega, \vec{k}_{\mathrm{F}}) \sim \frac{Z}{\omega + \mathrm{i}\Gamma}. \tag{5.4}$$

In this approximation the scattering rate Γ comes entirely from the spinon, hence it is half of Γ_Ω. The wavefunction renormalization constant comes as a product of a spinon and a holon contribution plus the effect of the quasi 1D structure of holons yielding for (the isotropic component of) Z:

$$Z \simeq (Q_{\mathrm{T}} m_{\mathrm{s}})^{1/2}. \tag{5.5}$$

By chiral invariance, physical absence of superconducting gap, the mass term induced by the spinon should be exactly cancelled by the holon–spinon short-range attraction. Notice that the same assumption underlines the recovery of the FS in the $SU(2)$ slave-boson approach (Lee *et al* 2006). We should emphasize that the electron FS, coinciding in our treatment with the holon FS, in PG is the 'small FS' centred at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$ with $k_{\mathrm{F}} \simeq \delta$ whereas in SM it is the 'large FS' with $k_{\mathrm{F}} \simeq 1 - \delta$.

In the PG phase besides (5.5) we have an additional angular dependent factor around, e.g., $(\frac{\pi}{2}, \frac{\pi}{2})$ given by $\frac{1}{2}[1 - \frac{1}{\sqrt{2}}[\cos(\arg \vec{k}_{\mathrm{F}}) + \sin(\arg \vec{k}_{\mathrm{F}})]]$ of the wavefunction renormalization yielding a reduction of the spectral weight outside the magnetic Brillouin zone, reminiscent of the 'Fermi arcs' of ARPES in underdoped cuprates (see Wen and Lee 1996, Lee *et al* 1998 for a similar situation in the $SU(2) \times U(1)$ slave-boson approach). This anisotropy is due to the matrix structure of the 'Dirac' Hamiltonian, describing physically the 'zitterbewegung', and the relation between the original spinless holon and the two species of 'Dirac holons' introduced by B_{MF}, which changes in the four quarters of the Brillouin zone centred at $(\pm\frac{\pi}{2}, \pm\frac{\pi}{2})$, thus recovering the standard translational invariance for the electron. We conjecture (Marchetti *et al*, in preparation) that an additional reduction of the spectral weight appears as a consequence of the attraction between spin vortices of opposite vorticity, neglected in the present MF, improving the agreement with ARPES data.

The structure exhibited by (5.4) shows that the gauge fluctuations are able to bind together spinon and holon into a resonance for low energies at the Fermi momenta, but with a strongly temperature dependent wavefunction renormalization constant. In particular $Z \sim T^{1/6}$, so Z vanishes if formally extrapolated to $T = 0$. This implies a peculiar non-Fermi liquid character for this system of 'electron resonances'. In fact, by gauge invariance the sum of spinon and holon currents is zero, but at $T = 0$ the spinons cannot move as they are gapped and without any possibility of thermal diffusion, therefore the electron system behaves as an insulator. A similar phenomenon is discussed within Weng's approach in Kou and Weng (2005). Existence of a FS is then compatible only because $Z = 0$. However, a real extrapolation to $T = 0$ cannot be done because of the range of validity of the approximation, as discussed above. The system therefore appears to fit naturally within the scheme of unstable fixed points (UFP) outlined by Anderson (2002).

6. Transport properties

In this section we sketch the results of calculation for physical transport quantities through Kubo formulae using the above approach and compare the theoretical results with the experimental data.

6.1. In-plane resistivity

The in-plane resistivity ρ is calculated via the Ioffe and Larkin (1989) addition rule: $\rho = \rho_s + \rho_h$, where ρ_s is the resistivity of the spinon-gauge subsystem and ρ_h that of the holon-gauge subsystem, which is subdominant and of standard form for a gauge-fermion system (Nagaosa and Lee 1990, Lee and Nagaosa 1992). Via the Kubo formula:

$$(\rho_s)^{-1} = 2\int_0^\infty dx^0 x^0 \langle j^s j^s \rangle (x^0, \vec{q} = 0), \tag{6.1}$$

where $\vec{j}^s \sim \partial\vec{\Omega} \sim Q_T \vec{\Omega}$ is the spinon current, and using (5.1) one obtains in PG:

$$\rho \sim \rho_s \sim \frac{|M|^{1/2}}{\sin(\frac{\arg M}{2})} \sim \begin{cases} T^{-1} & m_s^2 \gg \frac{cT}{\chi} \\ T^{\frac{1}{4}} & m_s^2 \sim \frac{cT}{\chi}. \end{cases} \tag{6.2}$$

From (6.2) one recovers the MIC and, due to the square root in M, an inflection point $T^* \sim \chi m_s^2$ at higher temperature, also found experimentally (Takagi *et al* 1992). Hence, in our approach the MIC is due to correlation effects, not to a disorder-induced localization, as discussed in section 2. In fact, within the range of validity for deriving (6.2) (above the Néel temperature and the 'spin-glass' temperature) the insulating behaviour is power-law like, not exponential. We believe that outside this range the qualitative behaviour should be the same, but the formula itself cannot be applied directly.

Moreover, the normalized resistivity $\rho_n = (\rho - \rho(T_{MIC}))/(\rho(T^*) - \rho(T_{MIC}))$ is a function only of the ratio $y = cT/T^*$: $\rho_n(y) = (1 + y^2)^{1/8}/\sin(\frac{1}{4}\arctan y)$. A universal behaviour of this quantity in fact can be verified in experimental data, and in general terms has already been empirically noted in publications (Wuyts *et al* 1996, Konstantinovic *et al* 2000). Furthermore, the dependence of $|M|$ on T^2 yields an approximate linear behaviour of in-plane resistivity in T^2, for temperatures just above the minimum in PG (Marchetti *et al* 2006), observed experimentally (Ando *et al* 2004), without involving the 'Landau' quasi-particles advocated in Ando *et al* (2004).

The introduction of a magnetic field perpendicular to the plane allows one to find MIC also for higher dopings suppressing superconductivity, and it yields a shift of the MIC to higher temperature which in turn produces a big positive transverse magnetoresistance (Marchetti *et al* 2001), as found in experiments in LSCO (Lacerda *et al* 1994, Kimura *et al* 1996, Abe *et al* 1999).

Non-magnetic impurities simply affect the resistivity by an upward shift as they appear via the Matthiesen rule only in ρ_h, consistent with irradiation experiments (Rullier-Albenque *et al* 2003). In contrast, Zn doping provides strong potential scattering changing the holon resistivity, and at the same time disturbs the AF background, making the AF correlation shorter, therefore qualitatively we expect a shift of MIC in spinon resistivity up in T, as observed in Zn-doped YBCO (Segawa and Ando 1999). This strong contrast of behaviour for non-magnetic (affecting the holon contribution to resistivity) and magnetic (affecting the spinon contribution) scattering, is indirect proof of the Ioffe–Larkin rule: *it is the resistivity, not the conductivity of the constituent slave particles* which is added to form the resistivity of the physical system.

On the other hand, in SM one obtains:

$$\rho_s \simeq 2\lambda Q_T^{-1}(\Gamma + m_s^2/\Gamma) = \frac{c'T}{\chi m_s^2}\lambda^2 + \frac{4m_s^4\chi}{c'TQ_T^2}, \tag{6.3}$$

where $\lambda \approx 0.7$. In the high-temperature limit $Q_T \gg m_s$, the damping rate in (6.3) dominates over the spin gap $2m_s$ and the spinon contribution to resistivity is linear in T, with a slope $\alpha \simeq (1-\delta)/(\delta|\ln\delta|)$. Lowering the temperature, the second term in (6.3) gives rise first to a superlinear behaviour and then, at the margin of validity of our approach, an unphysical upturn. The deviation from linearity is due to the spin gap effects and is cut off in the underdoped samples by the crossover to the PG phase. We expect that physically in the overdoped samples it is cut off by a crossover to a FL 'phase'.

The celebrated (approximate) 'T-linearity' can also be understood qualitatively as a consequence of Γ and the 'effectiveness' of the gauge fluctuations to form resonance, predominantly contributing to conductivity (see previous section), in a slab of momenta $Q_T \sim T^{1/3}$ around the Fermi surface. In fact the conductivity derived from the Boltzmann transport theory would be $\sigma_0 \sim \Gamma^{-1}$, but due to 'effectiveness' the physical conductivity is $\sigma \sim \sigma_0 Q_T \sim T^{-4/3}T^{1/3} \sim T^{-1}$. Similar considerations apply to the linearity found in SM for out-of-plane resistivity, spin relaxation time $^{63}(TT_1)$ and replacing T by ω of the AC conductivity (Marchetti *et al* 2005), as discussed below.

6.2. *In-plane IR electronic AC conductivity*

We compute the electronic AC conductivity in the two limits $\omega \ll T$, $T \ll \omega$, where ω is the external frequency. It turns out that up to the logarithmic accuracy one can pass from the first to the second limit by replacing T with ω in Q_T and Γ (we denote the obtained quantities by Q_ω, Γ_ω) and rescaling Γ by a positive multiplicative factor $\tilde{\lambda} \lesssim 1/2$. In PG, in the range $T \lesssim \omega \lesssim Jm_s$ the AC conductivity can be approximately obtained from ρ^{-1} derived from equation (6.2) substituting T with $\tilde{\lambda}\omega$. Hence it exhibits a broad maximum at a frequency ω_{MIC} (self-consistently $\ll Jm_s$), corresponding to a temperature slightly higher than T_{MIC}, as in fact experimentally seen in Dumm *et al* (2003). An *a*–*b* anisotropy of T_{MIC} and ω_{MIC} found in underdoped LSCO (Dumm *et al* 2003) would be generated naturally in the above scheme by an *a*–*b* anisotropy of the magnetic correlation length (Marchetti *et al* 2004b). The anisotropy of T_{MIC} strengthens our interpretation of correlation-induced MIC, since a disorder-induced localization is expected to have a unique MIC temperature. As T becomes greater than ω the AC conductivity becomes approximately ω independent.

In SM, for $\omega \ll T$ we have

$$\sigma(\omega, T) \sim \frac{Q_T}{\mathrm{i}(\omega - 2m_s) + \Gamma} \sim \frac{1}{\mathrm{i}(\omega - 2m_s)T^{-1/3} + T}, \tag{6.4}$$

while for $T \ll \omega$

$$\sigma(\omega, T) \sim \frac{Q_\omega}{\mathrm{i}(\omega - 2m_s) + \tilde{\lambda}\Gamma_\omega} \sim \frac{1}{\mathrm{i}(\omega - 2m_s)\omega^{-1/3} + \omega}. \tag{6.5}$$

From the above formulae the following features found experimentally in overdoped LSCO (Startseva *et al* 1999) and BSCO (Lupi *et al* 2000) are easily derived: besides the standard tail $\sim\omega^{-1}$, the effect of replacing Q_T by Q_ω is to asymmetrize the peak at $2m_s$ appearing in $\mathrm{Re}\,\sigma(\omega, T)$ for $\omega \ll T$ and to shift it towards lower frequency.

6.3. Spin-lattice relaxation rate $^{63}(1/T_1T)$

Assuming Q_T as the cutoff for the $|\vec{q}|$ integration and using the smoothness of the hyperfine field at this scale we derive in PG:

$$^{63}\left(\frac{1}{T_1T}\right) \sim (1-\delta)^2|M|^{-\frac{1}{2}}\left(a\cos\left(\frac{\arg M}{2}\right)+b\sin\left(\frac{\arg M}{2}\right)\right)$$
$$\sim \begin{cases} a+bT & m_s^2 \gg \frac{cT}{\chi} \\ T^{-\frac{1}{4}} & m_s^2 \sim \frac{cT}{\chi} \end{cases}$$

with $a/b \sim 0.1$. and the two terms are due to Re and Im of J_0 in (5.1).

Therefore we obtain a broad peak as observed in some cuprates (Berthier *et al* 1994) and up to a multiplicative constant a universality curve as a function of $y = cT/T^*$ can be derived. In SM one finds

$$^{63}\left(\frac{1}{T_1T}\right) \sim (1-\delta)^2\rho_s^{-1}. \tag{6.6}$$

Therefore, in the high-temperature limit we recover the linear in T behaviour for $^{63}(T_1T)$ and at higher dopings/lower temperatures the superlinear deviation, also found experimentally in overdoped samples of LSCO (Berthier *et al* 1997, Fujiyama *et al* 1997).

6.4. Out-of-plane resistivity

We assume an incoherent transport along the c-direction. ρ_c thus is dominated by virtual hopping between layers and it is calculated via the Kumar–Jayannavar formula (Kumar and Jayannavar 1992, Kumar *et al* 1997, 1998):

$$\rho_c \sim \frac{1}{\nu}\left(\frac{1}{\Gamma}+\frac{\Gamma}{t_c^2Z^2}\right), \tag{6.7}$$

where t_c in our approximation is an effective average hopping in the c-direction and ν the density of states at the Fermi energy. In PG the first term in (6.7) dominates, yielding an insulating behaviour:

$$\rho_c \sim (|M|\sin(\arg M))^{-1} \sim \begin{cases} T^{-1} & m_s^2 \gg \frac{cT}{\chi} \\ T^{-\frac{1}{2}} & m_s^2 \sim \frac{cT}{\chi}. \end{cases}$$

This crossover reproduces a knee experimentally found in Ito *et al* (1991), Yan *et al* (1995). A very strong decrease with T of the anisotropy ratio $\rho_c(T)/\rho_{ab}(T)$ in PG is also well reproduced by the above formulae.

In the SM phase the second, metallic, term in (6.7) dominates and substituting Γ and Z one recovers the T-linearity in the 'incoherent regime' $\Gamma \gg t_cZ$:

$$\rho_c \simeq \frac{J^2}{t_c^2m_s\nu}\frac{T}{\chi m_s^2} \simeq \frac{T}{(\delta|\ln\delta|)^{3/2}}. \tag{6.8}$$

A universal behaviour is also recovered for underdoped samples in the form $\rho_c = \alpha_c(\delta)\rho_c(y)$ with $\rho_c(y) \sim (1+y)^{-1/4}/\sin(\frac{1}{2}\arctan y)$ in PG, $\sim y$ in SM and the δ dependence of α_c is in agreement with the phenomenological proposal of universality (Su *et al* 2006).

7. Concluding remarks

As mentioned in the introduction, there is no consensus theory of HTS cuprates yet. What has been briefly summarized in this paper is a theoretical attempt to describe the metal–insulator crossover as the most spectacular phenomenon of cuprates in the pseudogap phase, as well as other transport properties of cuprates in the pseudogap and 'strange metal' phases in a self-consistent way, using the spin-charge gauge approach. To the best of our knowledge, this is the only available theoretical treatment of this crossover as an intrinsic property of the doped Mott insulators. We start from a two-dimensional t–J model in the limit $t \gg J$ to describe the strong correlation effects and use the gauge field approach to implement the single-occupancy constraints. In the mean field theory we have chosen, the spin degrees of freedom are represented by hard-core bosons, while the charge degrees of freedom are represented by spinless fermions, with a slave-particle gauge field binding them 'loosely'.

We have compared the calculated results with experiments, and in comparing the temperature-doping dependence of the in-plane resistivity, including the metal–insulator crossover (Marchetti *et al* 2000, 2001, 2004a), there are no adjustable parameters, except for the absolute scale of resistance. Moreover, the nontrivial doping/magnetic field dependence of the metal–insulator crossover temperature is a direct consequence of the theoretically predicted spinon mass gap $m_s^2 \sim \delta|\log\delta|$, where δ is the doping concentration, and the competition of the short-range antiferromagnetic order with dissipative motion of the charge carriers. Using the metal–insulator crossover as a clue to understanding the nature of the pseudogap phase, we could explain a number of peculiar transport properties in the pseudogap phase including the 'universality' and anisotropy of in-plane conductivity, the huge positive magnetoresistance in underdoped samples, the 'knee' in c-axis resistivity, the broad peak in the nuclear relaxation rate and the low-frequency spin fluctuation spectrum, within the same framework, using the same set of parameters.

Using the 'melting' of the π-flux lattice as the main signature of crossover from the pseudogap to the strange metal phase, we could easily generalize our approach to higher doping/temperature and calculate various transport properties to compare with experimental data in this phase. Apart from recovering the well-known linear temperature dependence of in-plane and out-of-plane resistivity, NMR relaxation rate, ω^{-1} asymptotic behaviour of the AC conductivity, etc, we could also account for some more subtle effects like the deviation from linearity and the asymmetry of the AC conductivity peak (Marchetti *et al* 2005).

Within the gauge field approach it is natural to consider the low-energy excitations as 'composite particles' consisting of slave particles loosely bound by the gauge field. A question arises: is this gauge field a pure mathematical construction or does it reflect to some extent the physical reality? In our opinion, the second is true. In strongly correlated systems the spin and charge degrees of freedom are neither 'completely separated' nor 'bound in a point-like particle', so the gauge field is an 'indispensable companion' of slave particles describing the charge and spin degrees of freedom. It took some time for condensed matter physicists to accept the quasi-particle as a real physical entity. The quasi-particle is a constituent of collective motion, and it ceases to exist as soon as it is taken away from the condensed matter. In a similar way, the gauge field disappears as soon as the strong correlation is switched off. In this sense, it is reasonable to consider low-energy excitations in strongly correlated systems as composite particles bound by the gauge field.

Acknowledgments

We would like to sincerely thank J H Dai, L De Leo, G Orso and F Ye for their collaboration at different stages of this project.

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